

Multiplexers, ALU Design

CS 64: Computer Organization and Design Logic

Lecture #13

Winter 2020

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Administrative

- Lab 6 due Thursday!
- Lab 7 will be posted today
- This Friday, the TAs will be in the lab
 - Attendance **is not mandatory**, but likely very useful for you to be there
 - Work on your Lab 7 b/c it's due next Tuesday

Lecture Outline

- More on Logic Simplification using Kmaps
- Multiplexers
- ALUs

Any Questions From Last Lecture?

Any Questions About the Lab?

5 Minute Pop Quiz!

- Given the following K-Map for binary function **F**:

<i>B</i> \ <i>AC</i>	00	01	11	10
0	1	1		1
1	1			1

- Group properly and write the optimized function **F**
- Draw the circuit

5 Minute Pop Quiz! (Solution)

- Given the following K-Map for binary function **F**:

		<i>AC</i>			
		<i>B</i> 00	01	11	10
<i>B</i>	0	1	1		1
	1	1			1

A Karnaugh map for a binary function F. The map is a 2x4 grid. The columns are labeled with AC values 00, 01, 11, and 10. The rows are labeled with B values 0 and 1. The cells contain 1s at (0,00), (0,01), (0,10), (1,00), and (1,10). A dashed red box highlights the 1s in the first row (B=0) and the 1s in the first and fifth columns (AC=00 and AC=10). Red arrows point left and right from the dashed box.

- a) Group properly and write the optimized function **F**

$$F = !A!B + !C$$

- b) Draw the circuit

See black board

Exercise 1

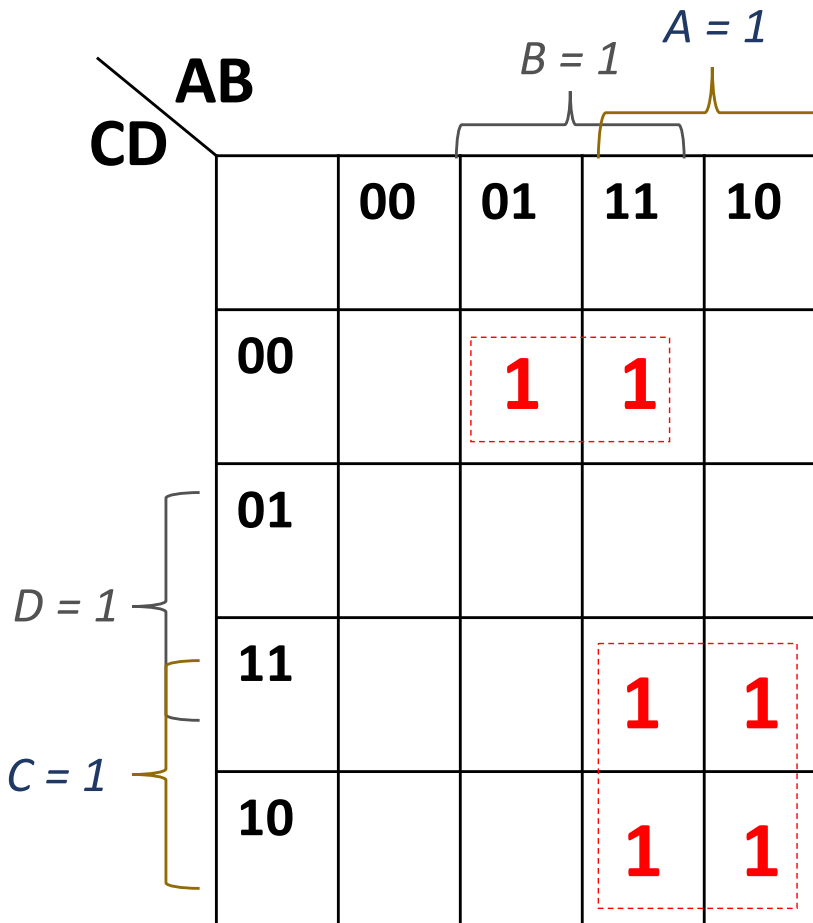
- Given the following truth table, draw the resulting logic circuit
 - **STEP 1:** Draw the K-Map and simplify the function
 - **STEP 2:** Construct the circuit from the now simplified function

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

Exercise 1 – Step 1

Get the simplified function

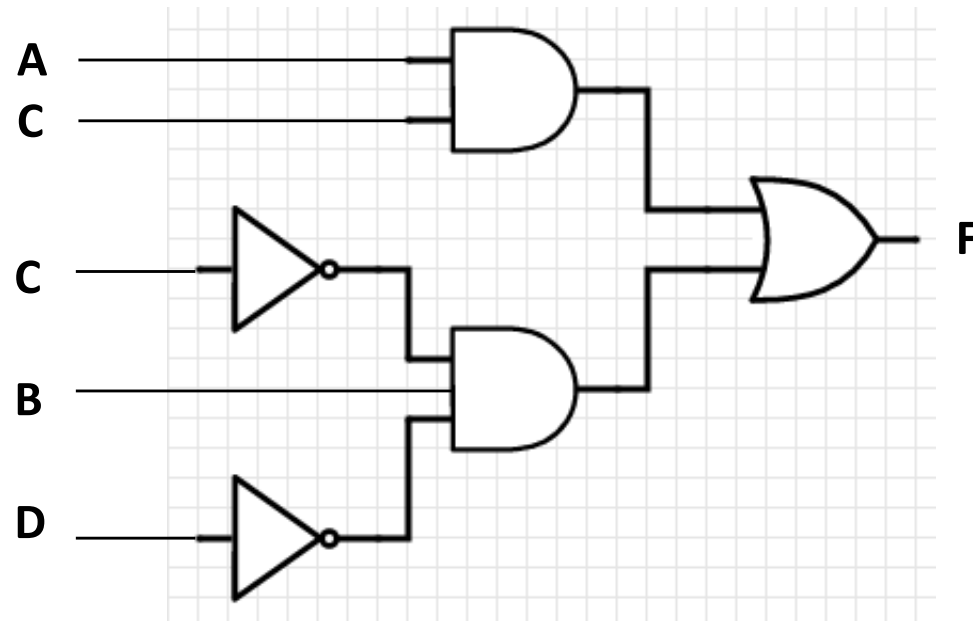


$$F(A,B,C) = B.\bar{C}.\bar{D} + A.C$$

Exercise 1 – Step 2

Draw the logic circuit diagram

$$F(A,B,C) = B \cdot \neg C \cdot \neg D + A \cdot C$$



Class Ex.

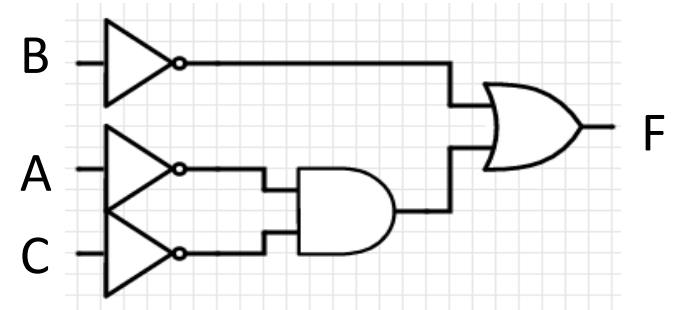
Exercise 2

- Given the following truth table, draw the resulting logic circuit

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

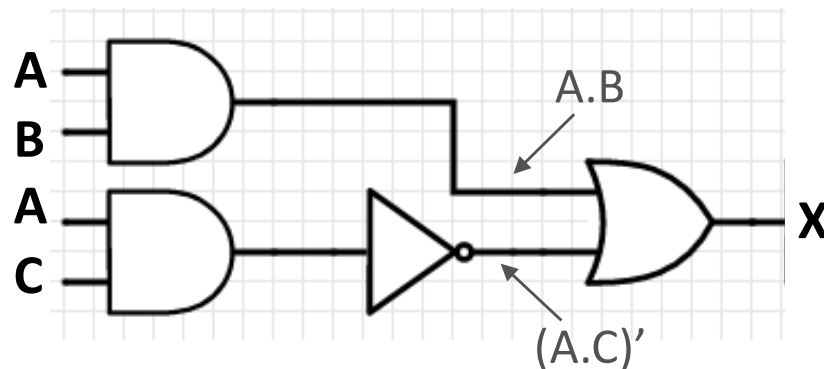
		AB			
		00	01	11	10
C	0	1	1		1
	1	1			1

$$F(A,B,C) = !B + !A!C$$



Exercise 3

- Given the following schematic of a circuit, (a) write the function and (b) fill out the truth table:



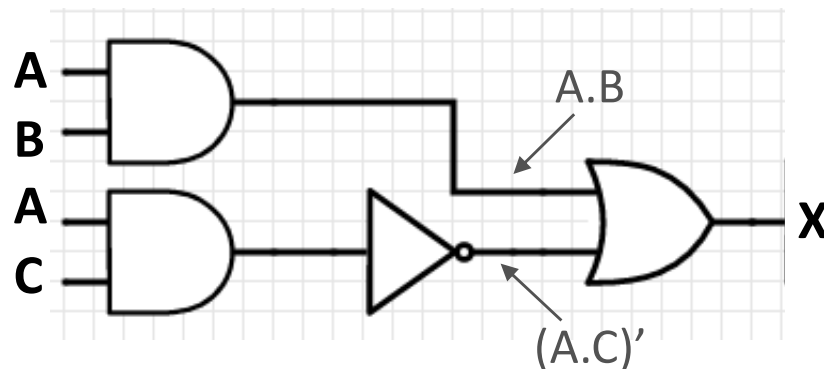
$$X = A.B + !(A.C)$$

(note that also means: $X = A.B + !A + !C$)

A	B	C	X
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Exercise 3

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$$X = A.B + !(A.C)$$

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A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

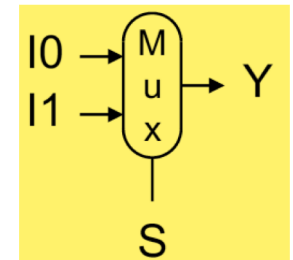
Multiplexer

- A logical selector:
 - Select either input A or input B to be the output

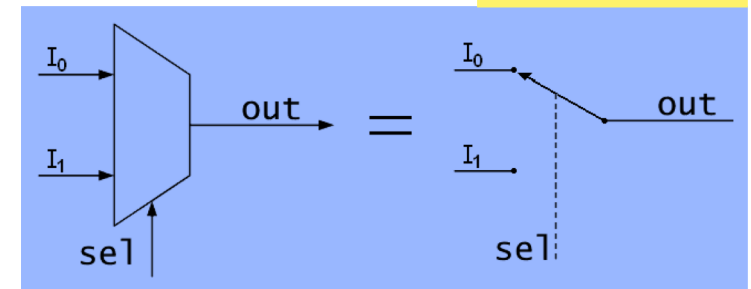
```
// if s = 0, output is a
// if s = 1, output is b
int mux(int a, int b, int s)
{
    if (!s) return a;
    else return b;
}
```

Multiplexer (*Mux* for short)

- Combinatorial circuits who function as a “chooser” between multiple inputs to be “driven to” the output
- Always multiple inputs (N), always ONE output (N-to-1 mux)
 - Can be drawn symbolically in 2 ways (trapezoid vs oval)
--- there’s NO difference, just a preference in drawing



- 1 of the input data lines gets selected to become the output, based on a 3rd “select” (sel) input
 - If **sel** = 0, then I_0 gets to be the output
 - If **sel** = 1, then I_1 gets to be the output
 - So: **OUTPUT** = $I_1 \cdot \text{sel} + I_0 \cdot \overline{\text{sel}}$



- The opposite of a Mux is called a **Demultiplexer** (or **Demux**)

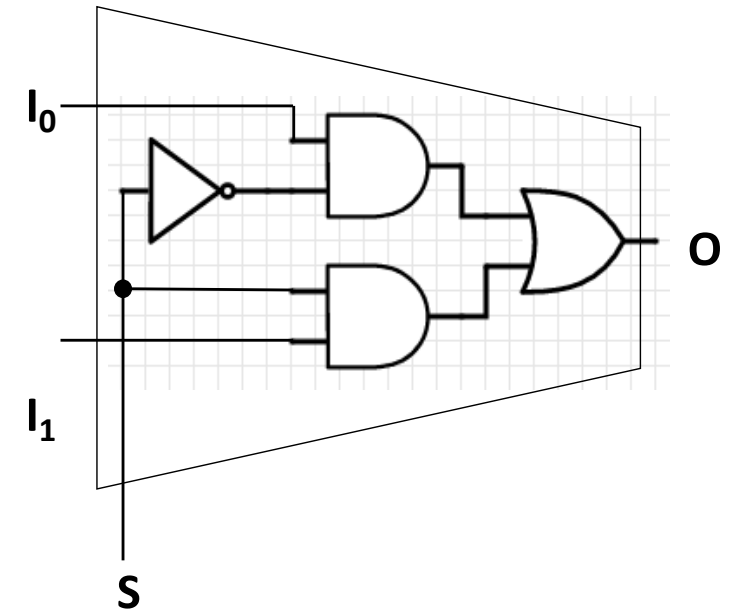
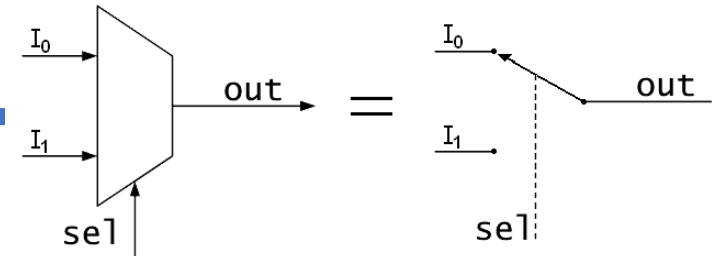
Mux Truth Table and Logic Circuit

1-bit Mux

I_0	I_1	S	O
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

S	$I_0 I_1$			
	00	01	11	10
0			1	1
1		1	1	

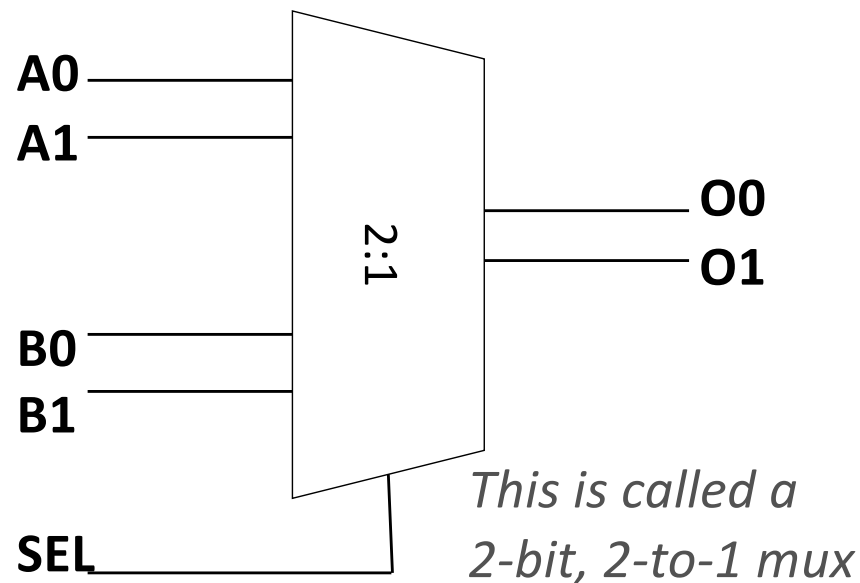
$$O = S \cdot I_1 + S' \cdot I_0$$



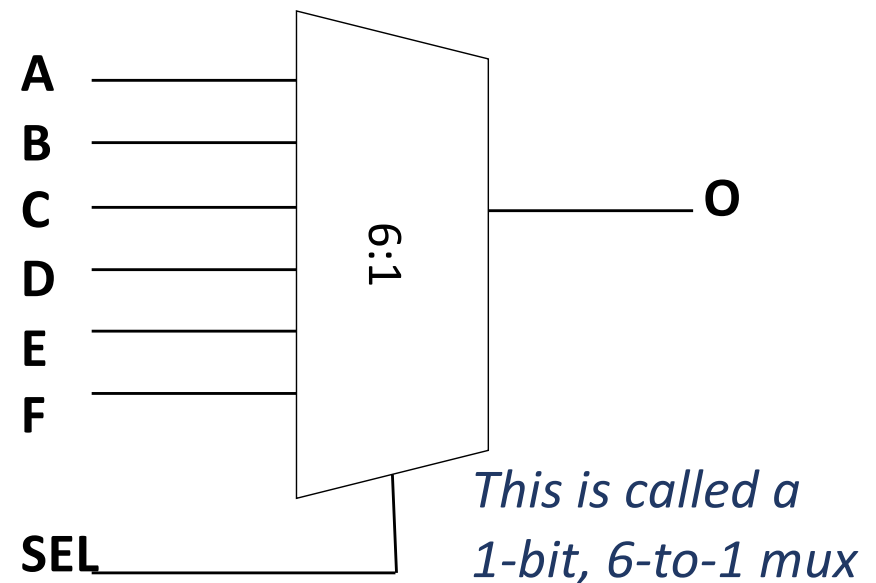
• = lines are physically connected

Mux Configurations

Muxes can have I/O that are multiple bits

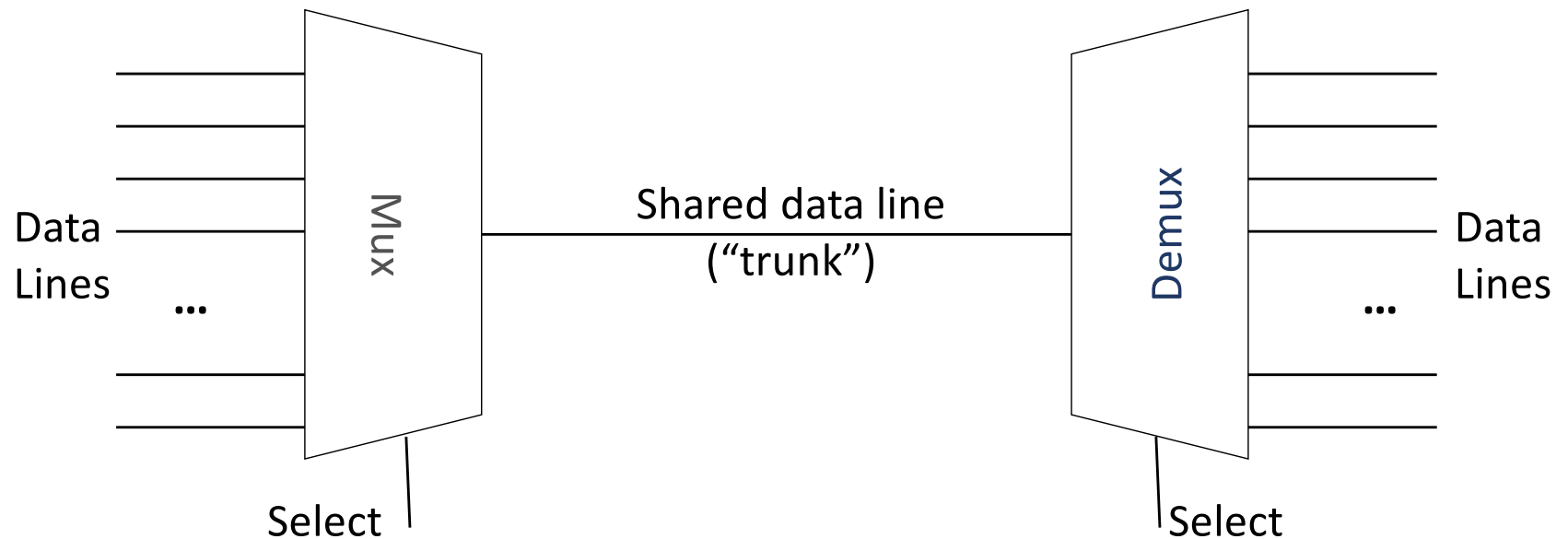


Or they can have more than two data inputs

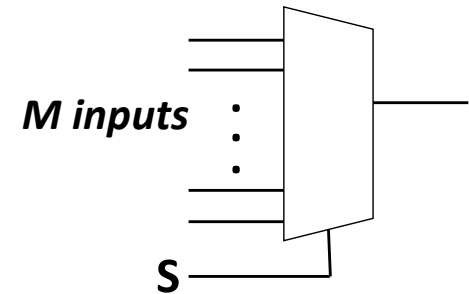


The Use of Multiplexers

- Makes it possible for several signals (variables) to share one resource
 - Very commonly used in data communication lines



Selection Lines in Muxes



- General mux description: **N-bit, M-to-1**
- Where: N = how “wide” the input is (# of input bits, min. 1)
 M = how many inputs to the mux (min. 2)
- The “select” input (S) has to be able to select **1 out of M inputs**
 - So, if $M = 2$, S should be at least 1 bit *($S = 0$ for one line, $S = 1$ for the other)*
 - But if $M = 3$, S should be at least **2 bits** *(why?)*
 - If $M = 4$, S should be ??? *(ANS: at least 2 bits)*
 - If $M = 5$, S should be ??? *(ANS: at least 3 bits)*

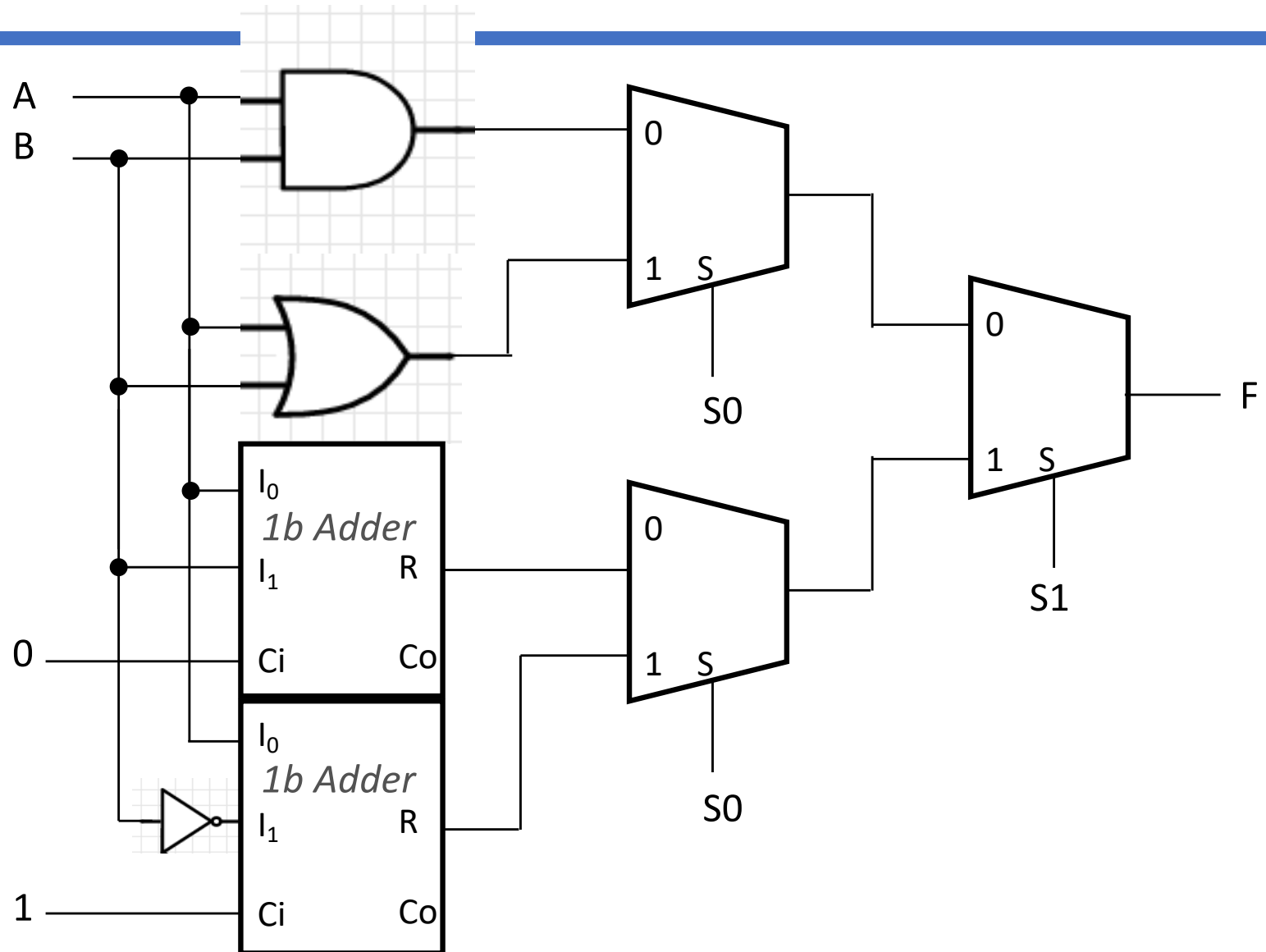
Combining Muxes Together

Can I do a **4:1** mux from 2:1 muxes?

Generally, you can do **$2^n:1$** muxes from 2:1 muxes.

What Does This Circuit Do?

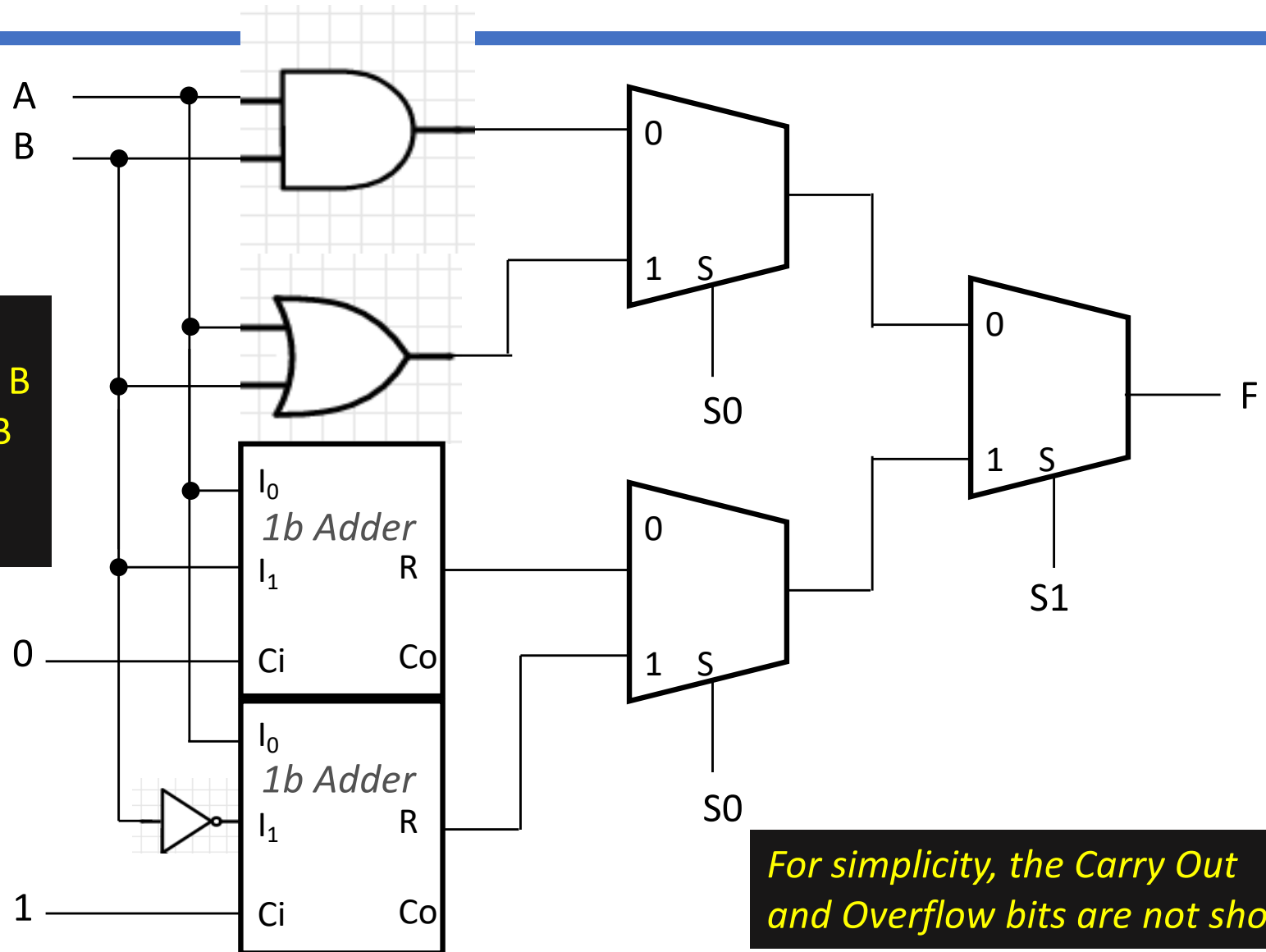
Class Ex.



Class Ex.

What Does This Circuit Do?

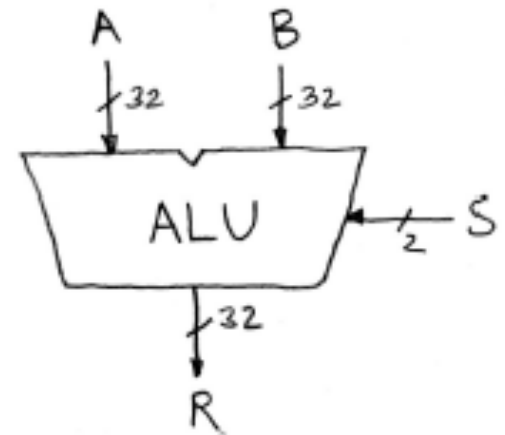
<u>S1</u>	<u>S0</u>	<u>F</u>
0	0	A && B
0	1	A B
1	0	A + B
1	1	A - B



For simplicity, the Carry Out and Overflow bits are not shown

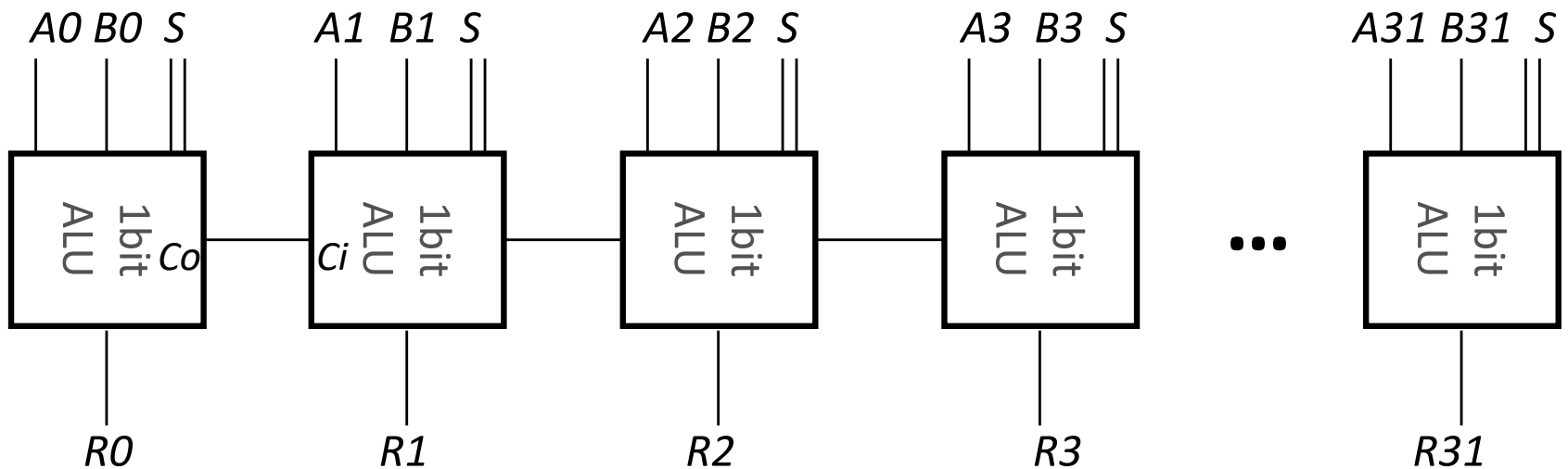
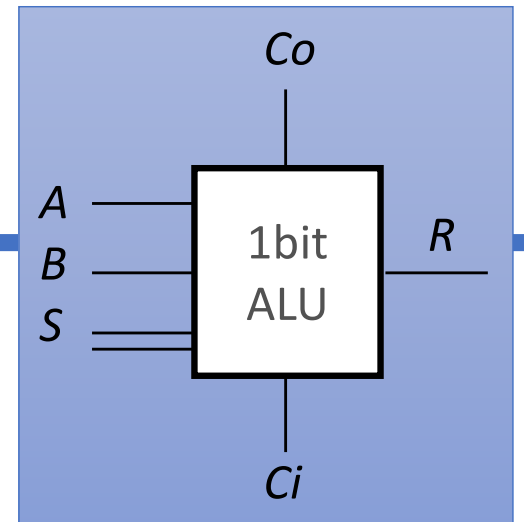
Arithmetic-Logic Unit (ALU)

- Recall: the ALU does all the computations necessary in a CPU
- The previous circuit was a simplified ALU:
 - When $S = 00$, $R = A + B$
 - When $S = 01$, $R = A - B$
 - When $S = 10$, $R = A \text{ AND } B$
 - When $S = 11$, $R = A \text{ OR } B$



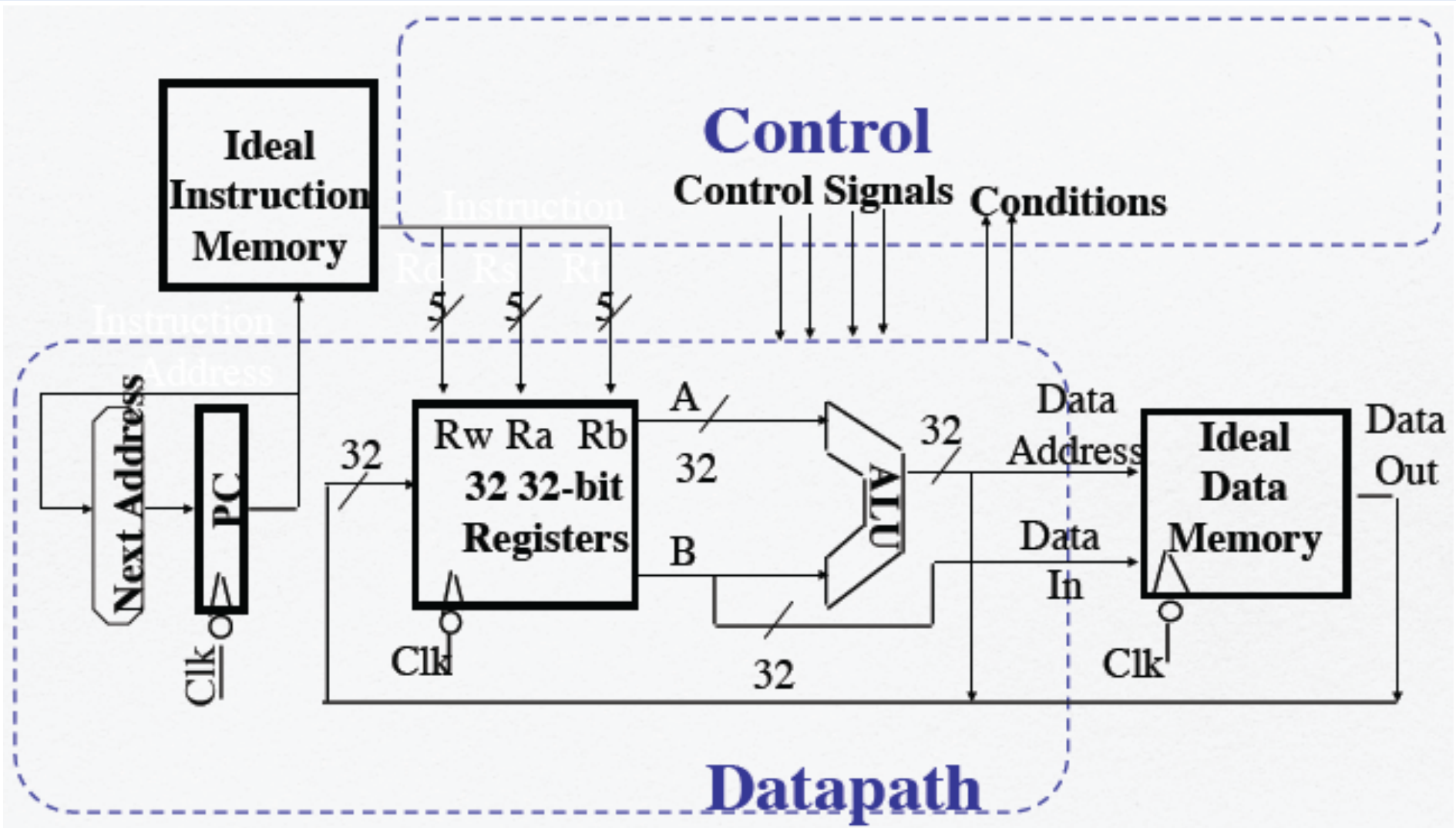
Simplified ALU

- We can string 1-bit ALUs together to make bigger-bit ALUs (e.g. 32b ALU)



Abstract Schematic of the MIPS CPU

Relevant to a future lab...



YOUR TO-DOs

- Go to Thursday lab

(we won't take attendance)

- Work on Lab 7, which is due **next Tuesday**

</LECTURE>